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Vaisala-Brunt frequency at an anchor station in the Florida current.

Paletta, Joseph

University of Miami

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THE UNIVERSITY OF MIAMI

VAISALA-BRUNT FREQUENCY AT AN ANCHOR STATION

IN THE FLORIDA CURRENT

BY

Joseph Paletta, Jr.

A THESIS

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Master of Science

Coral Gables, Florida

July, 1968

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Subject

Vaisala-Brunt frequency at an anchor station
in the Florida Current

Joseph Paletta, Jr.

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Vaisala-Brunt Frequency at an anchor station in the Florida Current.
(July 1968).   Abstract of a Master's Thesis at the University of
Miami.   Thesis supervised by Professor John C. Steinberg.

     The Vaisala-Brunt frequency is examined for data obtained at an
anchor station in the Florida Current for a ten day period.   There
are ninety-three separate S T D stations examined to yield a mean
profile of the Vaisala-Brunt frequency at this location.

     The maximum frequency of the mean profile is calculated to be
0.0169 sec-1 and occurs at a mean depth of 100 meters.   This cor-
responds to a Vaisala period of 6.19 minutes.

     The depth at which the maximum frequency occurs varies with
time, but with no apparent periodicity; this is also true of the
magnitude of the maximum frequency.   This randomness may be the
predominant feature of the Vaisala-Brunt frequency.

     The plot of frequency versus depth for each station reveals fine
structure in the vertical column, showing a layered structure.   The
shallower layers appear to have a stronger lamination effect on the
density structure.

ACKNOWLEDGEMENT

and friendship through these two years.

The ultimate in appreciation goes to my wife, Cynthia, whose support, patience, understanding, and enthusiasm smoothed the road to the completion of this thesis.

Lt. Joseph Paletta, Jr., USN

Coral Gables, Florida

July 1968.

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1. INTRODUCTION

Density gradients affect the motion of the seas. The salinity, temperature and pressure gradients all cause the sea water density to vary with depth. Most of the density gradient is caused by the compression of the water, the rest by the variation of temperature and salinity with depth (Eckart, 1961). Associated with the density gradients are well defined density boundaries which exhibit certain frequencies and modes of oscillation. Thus, any given density boundary will have its own normal oscillating frequency. This is called the Vaisala-Brunt frequency and is expressed in radians per second. It is a measure of the stability of oceanic stratification (LaFond and LaFond, 1966). The distribution of this frequency is one of the most important dynamical features of the ocean.

The concept of vertical stability in the ocean was expressed by Sverdrup, et al., in 1942 as $E = 10^{-3} \dfrac{d\sigma_t}{dZ}$, and appears in Richardson's number as $\dfrac{1}{[\rho]} \dfrac{d[\rho]}{dZ}$. Pollack (1954) defines vertical stability as follows:

1. As a measure of the restoring force (or resistance to change) in the vertical direction.

2. As an acceleration per unit change of geometrical depth; the acceleration may be expressed per unit change of pressure also; Hesselberg used this approach.

3. As an expression of any continuous function which is a parameter of the resistance to deformation of the fluid column.

4.   As the amount of energy required to move a particle of unit

mass a unit displacement.

In definitions 2 and 3, stability is a function of the difference

between an observed vertical mass distribution and that of a neutral

equilibrium mass distribution. Therefore, zero corresponds to the

neutral position, stability is indicated by the positive values and

instability by negative values (Pollak, 1954).

According to Hesselberg, stability $E = \frac{g}{\rho} \frac{d\rho}{dZ}$, or $E = \frac{g}{\rho} [\frac{d\rho}{dZ} - (\frac{d\rho}{dZ})_A]$,

where $(\frac{d\rho}{dZ})_A$ is the density gradient of neutral equilibrium or the mass

distribution in a column of homogeneous water (pressure=0, salinity and

temperature=constants) with negligible molecular conductivity. Bjerknes,

et al., (1934) noticed the adiabatic density gradient $(\frac{d\rho}{dP})_A$ was the re-

ciprocal of the term in the Lapalcian expression for the velocity of

sound $C^2 = \frac{(dP)}{(d\rho)_A}$, (Pollak, 1954). Therefore, the adiabatic density

gradient can be replaced by $1/C^2$ (see Appendix A). This is possible

because of the analogy between the pressure, p, and the geometric unit

of depth, Z. Pollak equates stability in the geometric units as:

$E = \frac{9.81}{\rho} [\frac{d\rho}{dP} - \frac{9.81\rho}{C^2}]$, which appears similar to an expression devised

by the meteorologist Vaisala in 1928, which he designated N^2, a sta-

bility expression with dimensions of frequency. Eckart mentioned this

expression in notes of a seminar given in 1949. According to Pollak,

the source for the expression was a lecture by Solberg at the Univer-

sity of California in 1946. The original reference could not be located.

We can examine the frequency, N, by noticing that if the density

of a fluid is changed adiabatically, then $dP = C^2 d\rho$. Where dP and $d\rho$

are the changes in pressure and density, and C is the speed of sound.
If dP is caused by changing the level of a parcel of water by an
amount dZ then $dP = -\rho g dZ$ if we assume a positive Z - axis up, and g
is the acceleration of gravity.

Elimination of the term dP gives:

$$\frac{1}{\rho} \frac{dP}{dZ} = \frac{-g}{C^2}$$

The quantity g/C^2 is identified as the adiabatic density gradient and
is that part of the actual density gradient associated with compres-
sibility effects. Let $E = -(\frac{1}{\rho} \frac{d\rho}{dZ} + \frac{g}{C^2})$ and we find that, generally
speaking, it is a positive value in the seas. The quantity N^2 in-
troduced by Vaisala and Brunt is equal to gE. It has frequency
dimensions and therefore the period, $T = \frac{2\pi}{N}$ (Eckart, 1961).

In considering the significance of the coefficient N, Eckart
(1960) considers the oscillation of a small fluid mass displaced from
its zero position and allowed free, adiabatic movement, with the
positive Z-axis upward. He assumes the particle of fluid is in a
small balloon-like membrane which permits changes in density to occur
unimpeded, and thereby maintain the outside and inside pressures the
same. The displacement causes a pressure change and an accompanying
density change within the balloon, so that $d\rho$ inside $= \frac{dP}{C^2}$. The out-
side density change will be :

$d\rho$ outside $= \frac{\xi}{\rho} \frac{dP}{dZ}$, where ξ is the displacement. The buoyant
force on this balloon is then equal to

$$g \, (d\rho \text{ outside} - d\rho \text{ inside}), \quad (1)$$

which is

$$g \, \zeta \left(\frac{d\rho}{dZ} - \frac{\rho g}{C^2} \right) = -\rho \, N^2 \, \zeta, \qquad (2)$$

where

$$N^2 = -g \left[\frac{1}{\rho} \frac{d\rho}{dZ} + \frac{g}{C^2} \right] . \qquad (3)$$

If the frequency, N, is real, the parcel of water in the balloon would oscillate freely at the frequency N. Instability would exist if N^2 was negative, causing the ballon to leave its initial position (Eckart, 1960).

It is believed that N is the limiting frequency for the free internal waves because of viscous damping (McLellan, 1965). Internal waves can exist if their frequency of oscillation is greater than the inertial frequency, but less than the maximum Vaisala frequency, N_{max}. There should, therefore, be no free internal waves with frequencies above N_{max}, and few with frequencies above the local Vaisala frequency in a particular water mass (Cox, 1962). In examining internal waves near Bermuda, Haurwitz, et al., (1959) used the Vaisala frequency. They associated the maximum frequency with the seasonal thermocline and the second maximum found at a deeper depth, with the permanent thermocline. They also concluded that at any depth, there should be no frequency larger than the maximum Vaisala frequency. Hence, this may be designated the "cut-off" frequency. This cut-off frequency is known to vary with time and space and must be computed for each vertical density distribution. When converted to period the Vaisala frequency relates to the natural oscillation governed by the stability of the water layers (Lee, 1960).

In the upper four thousand meters of the ocean the period has a wide range, from one minute to four hours. In deeper waters the

frequency is difficult to compute because the terms in the brackets
of the equation are very nearly equal and of opposite sign. At the
temperatures and pressures found in the deep waters the values of the
coefficient of thermal expansion are doubtful, and at best are extra-
polated values. These extrapolated values create large uncertainties
in the value of Vaisala frequency. Uncertainty also arises from
errors in measuring small salinity gradients (Eckart, 1961).

An investigation of the Vaisala frequency in the Florida Current
was conducted using pilot data collected in a twenty-four hour period
during the winter of 1966 by Dr. Saul Broida. Two locations were used
with stations taken alternately between them. Density parameters
were measured in situ with a Bisset-Berman Salinity-Temperature-Depth
(STD) system. The data were analyzed at fifteen meter depth increments
from the surface to seven hundred meters. The Vaisala frequency and
period were computed for each depth increment, and plotted versus
depth.

The frequency equation as stated by Eckart was used:

$$N^2 = -g \left(\frac{1}{\rho} \frac{d\rho}{dZ} + \frac{g}{C^2} \right) \quad \text{in radians per second.}$$

A computer program was devised to calculate the terms of the equation
and solve for the frequency and period, where $T = \frac{2\pi}{N}$. Curve fit-
ting was accomplished by a cubic spline interpolation (see Appendix B).

Analysis of the resulting pilot data disclosed some interesting
features:

1. At any given depth the frequency varied with time. Some
   periodicity was implied, but insufficient data points did
   not allow any definite conclusions.

2. Minimum period values occurred at the boundary of the
    isothermal layer and the mixed upper layer.

3. The shape of the frequency curves indicated a multi-layered
    structure of laminations of density, as noticed elsewhere
    in the oceans by Stommel and Fedorov (1967), and others.

4. The depth of maximum frequency varied with time.

II. METHODS

A. DATA COLLECTION

During the period from 19 February to March 1, 1968 the R/V PILLSBURY was anchored in the Florida Current in 460 fathoms of water at Latitude 25-39.1 N Longitude 079-48.0 W (Fig. 1). The objective was to gather temperature, salinity, and depth information to study density fluctuations in the Florida Current, and analyze the Vaisala-Brunt frequency at this location. This cruise, P6801, was the longest continuous anchor station completed in the Florida Straits.

A Bissett-Berman Salinity-Temperature-Depth probe (STD) system was utilized to gather the _in situ_ data. Probe casts were conducted at two hour intervals. The initial lowerings were made to 750 meters, but an unfortunate wire entanglement later limited the casts to a maximum 500 meter depth. The STD probe utilizes sensing elements which send back the _in situ_ data as a frequency signal for each parameter. These frequencies are recorded on graphic plots and magnetic tape. The conversion to actual values of depth, temperature, and salinity is done with standard conversion tables of frequency versus the parameter measured. To maintain the calibration and accuracy of the system, Niskin bottle samples were taken for comparison. At selected times a sample was taken at the surface and one at the maximum depth along with the probe lowering. The salinity and temperature readings of both instruments were compared and found to be in good agreement.

Because of the nature of the Florida Current a strong surface current prevailed throughout the experiment, approximately 4.2 knots. Technical problems and rough seas limited the total number of stations taken to ninety-three during the ten day period.

The ship's position was determined by visual bearings and radar ranges to Fowey Rocks Light. The accuracy of the anchor position is ± 100 meters in an East-West direction; ± 400 meters North-South.

Some gaps do exist in the sequence of data collection which are attributed to mechanical failures and weather conditions. However, the major portion of the stations are at two hour intervals with the last sequence of stations, number seventy-one to ninety-three, being taken at hourly intervals.

FIGURE 1.   Chart of ship's position
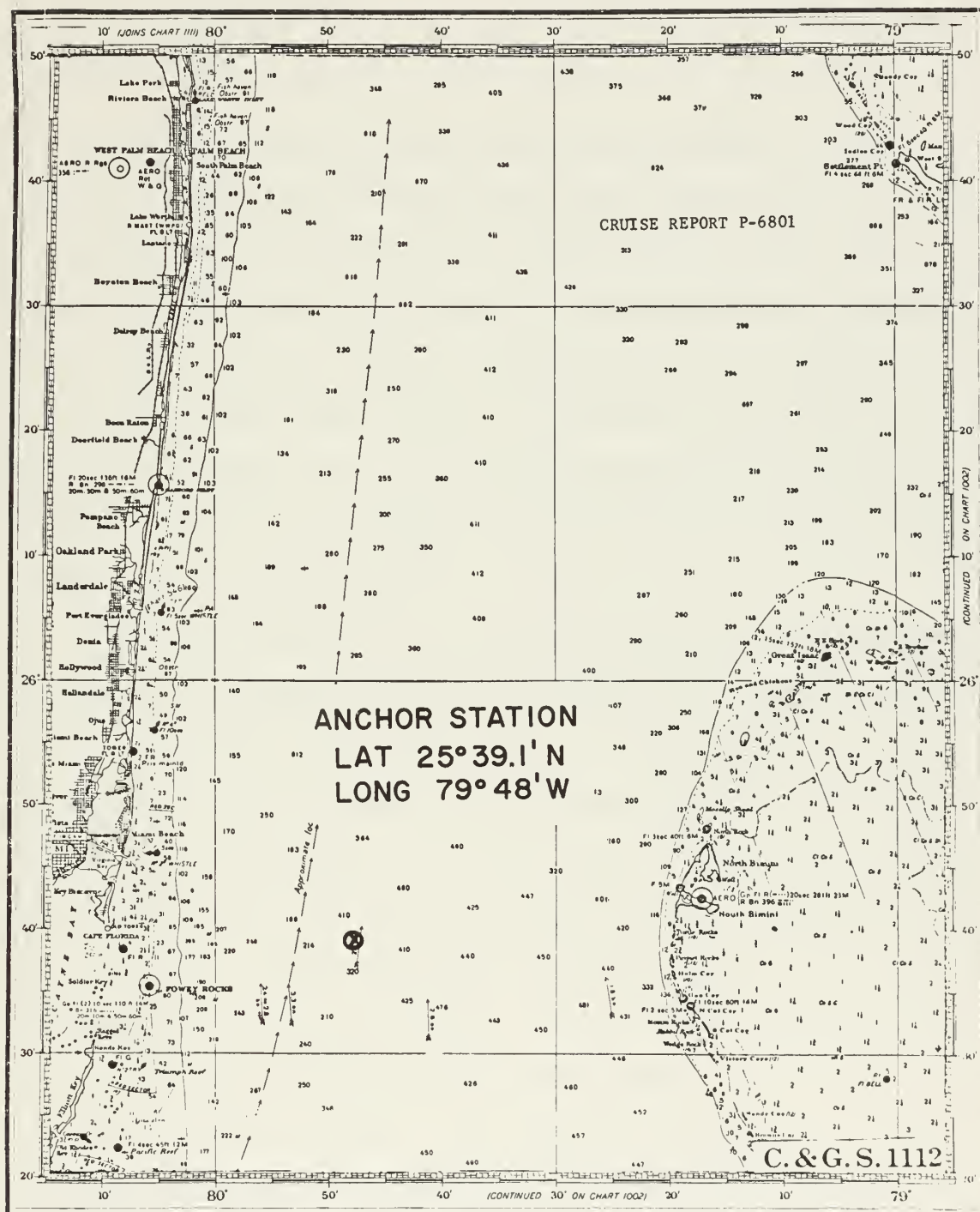
B.   DATA ANALYSIS

The STD data from the cruise were in analog format for each station, but those stations taken after station number 32 also recorded the parameters on magnetic tape via a digital data log unit (DDL). Those stations not recorded on magnetic tape initially were digitized with a CALMA converter. This is a manual trace of the temperature and salinity graphs, which records the horizontal and vertical trace distances on magnetic tape. By computer conversion the distances are changed to the desired parameters. The end result is a magnetic tape record of all the station data.

The STD scanning rate was set at 0.1 seconds giving a high density of points on the traces and tapes. To select the most representative value of a parameter at a given depth an averaging process was used. The DDL values were averaged at twelve meter intervals from the sur- face to the maximum depth of the station cast. A comparison between the averaged values and the actual graphic values of temperature and salinity at given depths indicated a tolerance of $\pm$ 0.02° C for tempe- rature and $\pm$ 0.03 o/oo for salinity. The averaging process provided the best values for analysis for the total cast.

The Temperature-Salinity (T-S) diagram for the composite of the data was investigated and compared to the historic data for that region. The historic data was taken from the works of Parr and various cruises conducted by the Institute of Marine Sciences. Even though the curves were geometrically coincident, the salinity values of some stations

appeared to be lower than the historic data. Although the spread of
the T-S curve was larger than might be expected, the relative changes
in temperature and salinity appear adequate for analysis of the gra-
dients. Thus, these relative values are acceptable for the investi-
gation of the Vaisala frequency which is density gradient dependent.

There are many small features shown on the temperature traces
which are probably real, but the salinity traces have many small scale
spikes of low salinity which appear false, probably caused by imperfect
matching of the time constants of the temperature and conductivity
sensors of the STD. The falsification appears as fine scale variations
in salinity at the points of strongest vertical temperature gradient.
The deeper the traces go, the more the salinity trace becomes a sharp
indicator of temperature gradients.

It is felt that many of these spikes are compensated for in the
averaging process and also in the interpolation of points while curve
fitting.

C.   COMPUTATION

Once the station data was compiled in terms of temperature,
salinity, sigma - t, and specific volume anomaly, for each twelve meter
increment of depth, the parameters for solving the Vaisala-Brunt fre-
quency equation were avialable. The value for the acceleration of
gravity was assumed constant at 9.7893 meters per second at Latitude
25.7 N. Sound speed was computed at each depth increment by the
following equation:

   $C = 141,000 + 421\ t - 3.7\ t^2 + 110\ S + 0.018\ d$, in (cm/sec),   (4)

where

   t = temperature (°C), S = salinity (o/oo), d = depth (cm).
These values of speed were converted to meters per second.

   The in situ density was computed and used in determining the
quantity $\dfrac{d\rho}{dZ}$. The equation used to find the density was:

   $OC\ (35,0,P) + \delta = OC$ in situ (cm^3/qm),   (5)

where

   $OC\ (35,0,P)$ = standard specific volume at depth, and δ = specific
volume anomaly. The reciprocal of the specific volume in situ is the
in situ density, ρ . The standard specific volume was computed by
linear interpolation. The values decreased linearly with depth by
$45 \times 10^{-7}\ cm^3/$ gm/ meter.

   The values of density in situ were plotted versus depth for each
station and a cubic spline interpolation fit was made to the points,

(Fig. 2). Using interpolated values from these curves the slopes,
$\frac{d\rho}{dZ}$, were found at five meter intervals, as were the values of density
at those depths.

The frequency equation was solved using these interpolated values
in the equation:

$$N^2 = -g \left(\frac{1}{\rho} \frac{d\rho}{dZ} + \frac{g}{C^2}\right). \tag{6}$$

Frequency was converted to period by $T = \frac{2\pi}{N(60)}$ minutes. The frequency
was plotted versus depth and analyzed, (Fig. 3). A mean frequency
curve was also plotted for the entire cruise data by averaging the
interpolated values of frequency at five meter increments for the
ninety-three stations (Fig. 4). Where negative values of N^2 were
encountered, they were eliminated from the average to present only
positive values of frequency. These negative values are interesting
in that if they really exist they could represent instability in the
laminar structure of the water column, but they may be caused by
erroneous data readings as a result of time lags in the sensors. It
would be interesting to investigate these in future studies. The
historic data from the Florida Current indicate strong stability in
the area of the experiment; therefore, it is felt that these negative
values might really be neutral stability points, approaching N=0.
Because they were few and variable in depth, it is felt that the
averaging process was not prejudiced by their elimination, nor was the
investigation of the frequency in depth and time. When possible, depths
were chosen where a minimum or no negative values of N^2 were noted
during analysis.

To help diminish errors in using Vaisala frequency as a measure
of stability, small depth increments were chosen for analyzing gradients

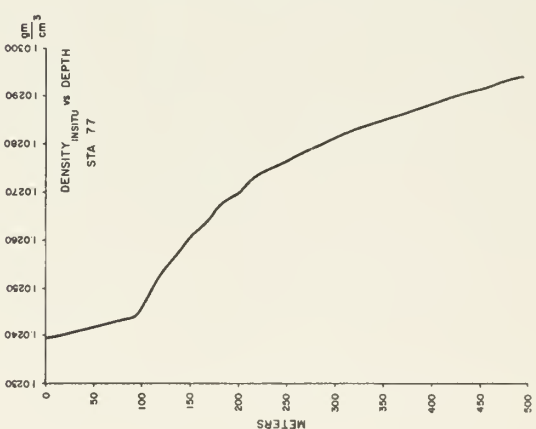15

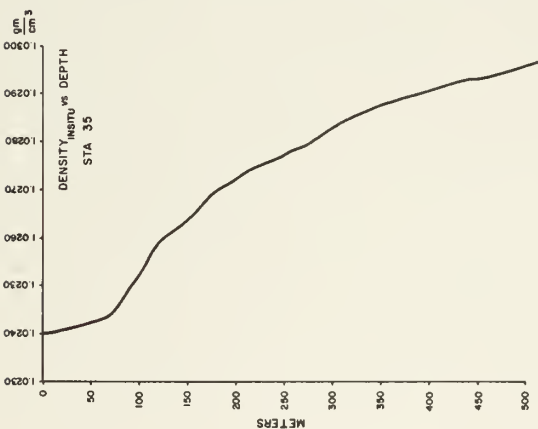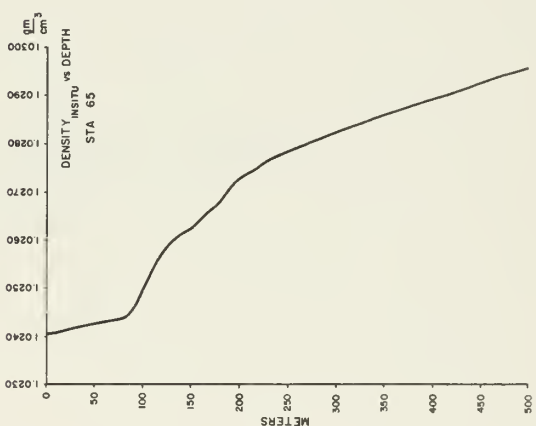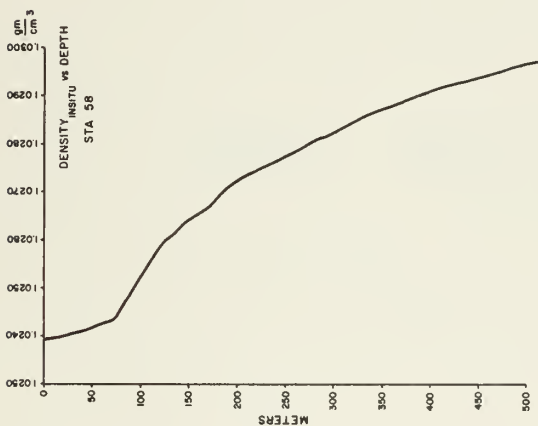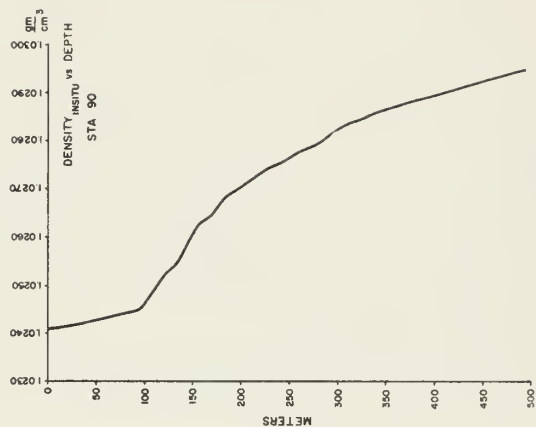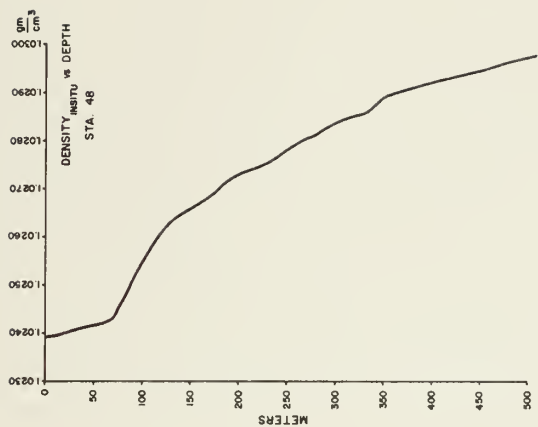FIGURE 2.   Density versus depth profiles

(6 stations)

FIGURE 3.   Frequency versus depth profiles

(6 stations)
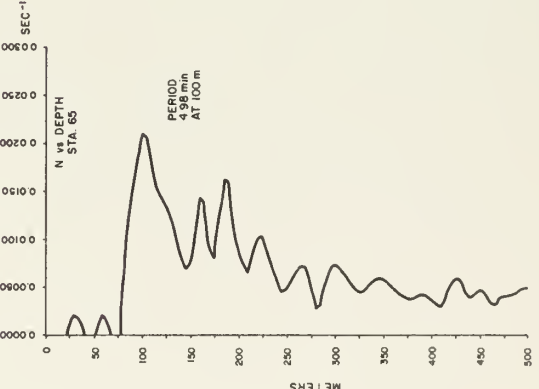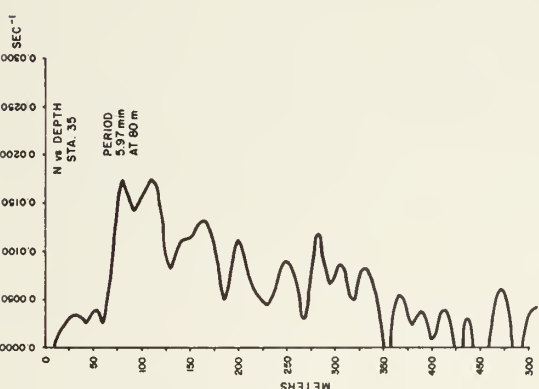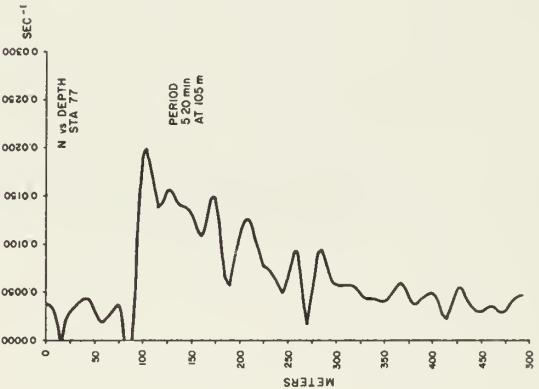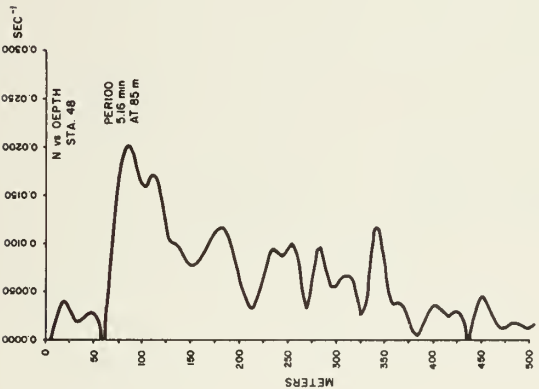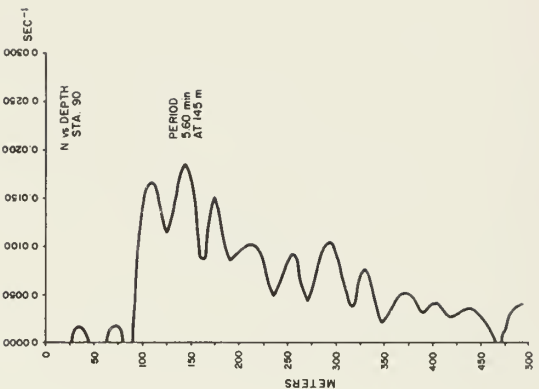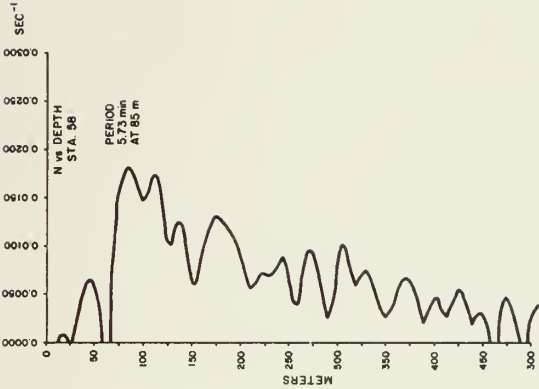
FIGURE 4.   Mean frequency versus depth profile

$N_{AVG.}$ vs DEPTH

PERIOD
6.19 min.
at 100 m.
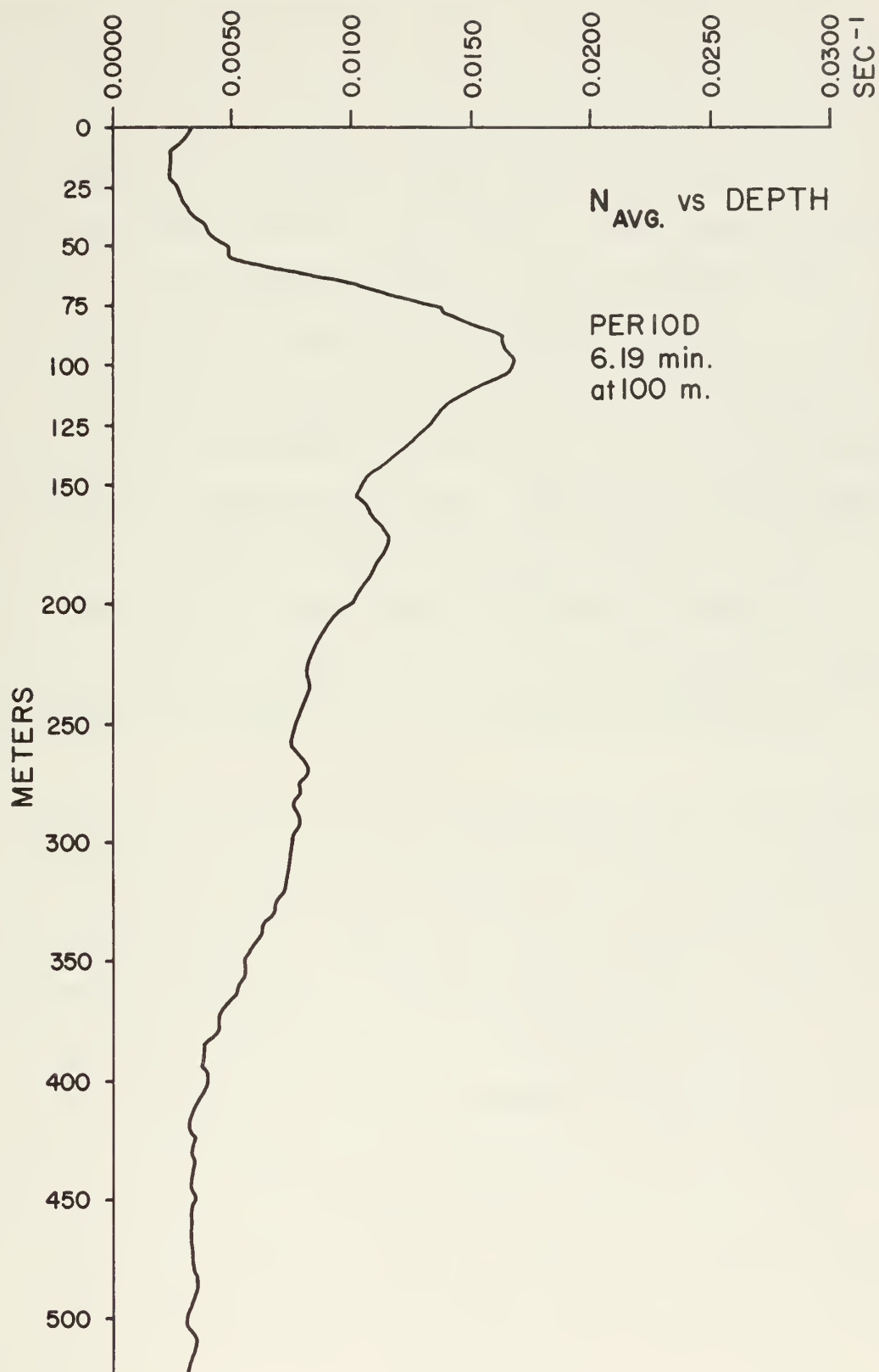
in density. Hence, the five meter increment for interpolated values
of density. According to Roden (1968) it is believed that any fun-
damental oscillations would differ from the Vaisala frequency by about
twenty-five per cent for this vertical displacement. In studying the
Vaisala frequency in the Gulf of Alaska, Roden found that the non-
linear cases showed stability oscillations no longer symmetrical about
the mean, as is the Vaisala oscillation, and the fundamental fre-
quency of oscillation differed from the Vaisala frequency.

In computing the standard deviation and variance of the fre-
quency for the entire period of observations, 100, 245, and 350 meters
were selected as representative depths. They were so chosen because
they contain a minimum of negative N^2 values, which were to be ex-
cluded; they included the mean depth of the maximum frequency as
compiled in the mean frequency curve; and they are spaced far enough
apart to give an in depth view of the frequency on a statistical
basis.

From each station the maximum frequency was gathered and plotted
versus time (Fig. 5), and so was the depth at which the maximum fre-
quency occurred (Fig. 6). These were visually inspected for some
recognizable periodicity, but none was readily apparent. In fact,
the random nature of the function is the primary observation.

From the mean profile of Vaisala frequency a maximum value of
0.0169 sec^{-1} was obtained, which corresponds to a period of 6.19
minutes. The depth of the maximum frequency was 100 meters. The
computed standard deviation of depth for maximum frequency was ± 15.2
meters, with a relative dispersion (coefficient of variance) of 16.6
per cent.

The standard deviation of the frequency at 100 meters was $\pm$ 0.0037 sec^{-1}, with a relative dispersion of 22.8 per cent. At 245 meters the standard deviation was $\pm$ 0.0030 sec^{-1} and the relative dispersion was 40.5 per cent. For 350 meters the standard deviation was $\pm$ 0.0031 sec^{-1} with a relative dispersion of 60.9 per cent.
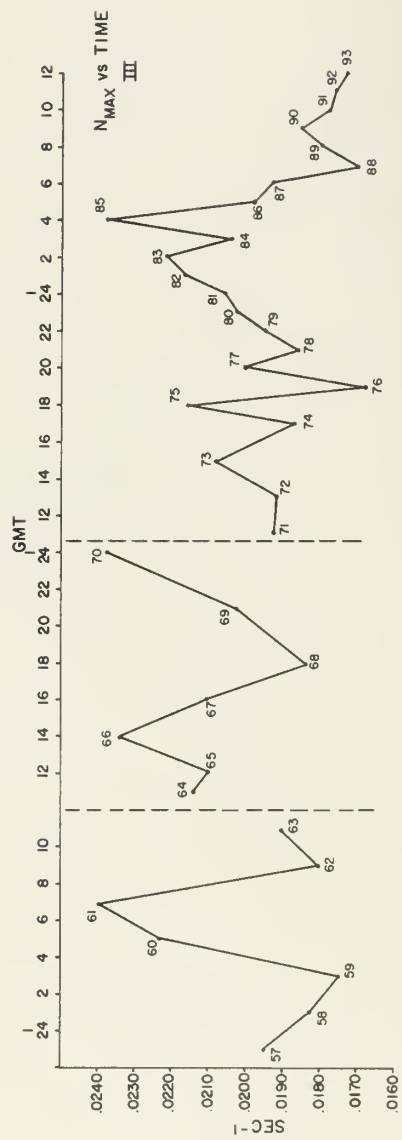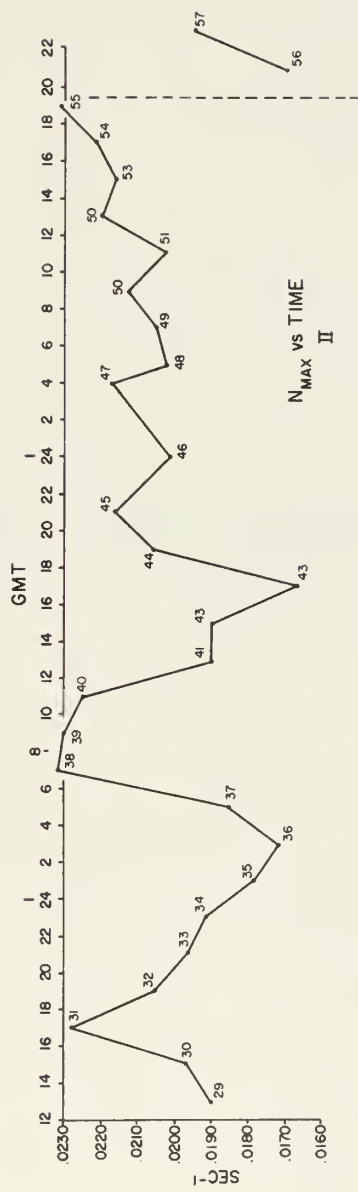
FIGURE 5.   Maximum frequency versus time.

N_MAX vs TIME I

N_MAX vs TIME II

N_MAX vs TIME III
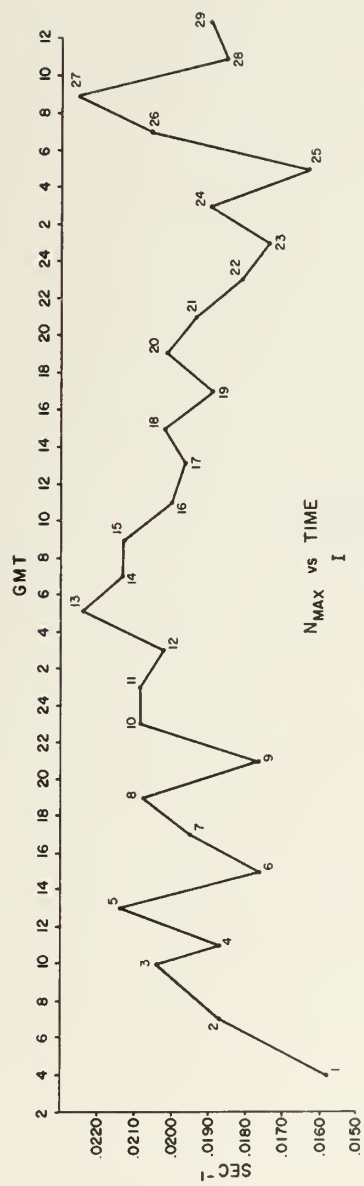
FIGURE 6.   Depth of maximum frequency

versus time

DEPTH OF N_{MAX} vs TIME
I

DEPTH OF N_{MAX} vs TIME
II

DEPTH OF N_{MAX} vs TIME
III
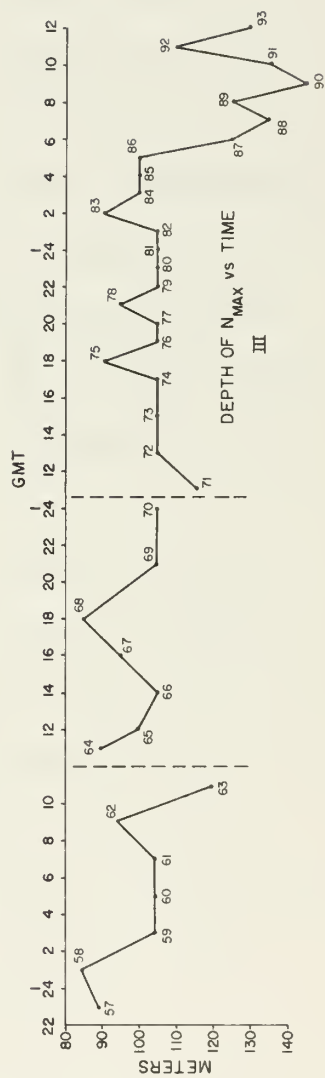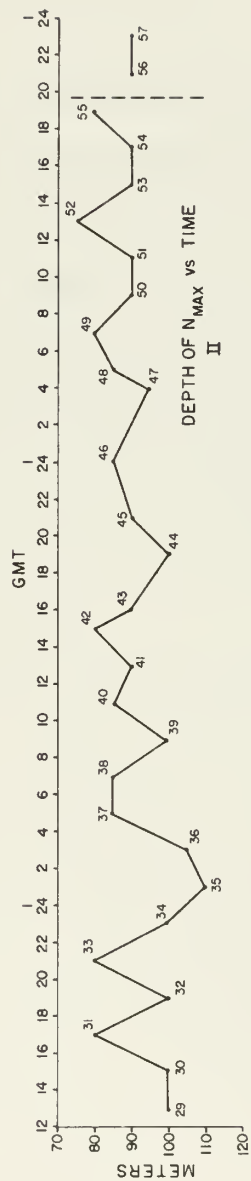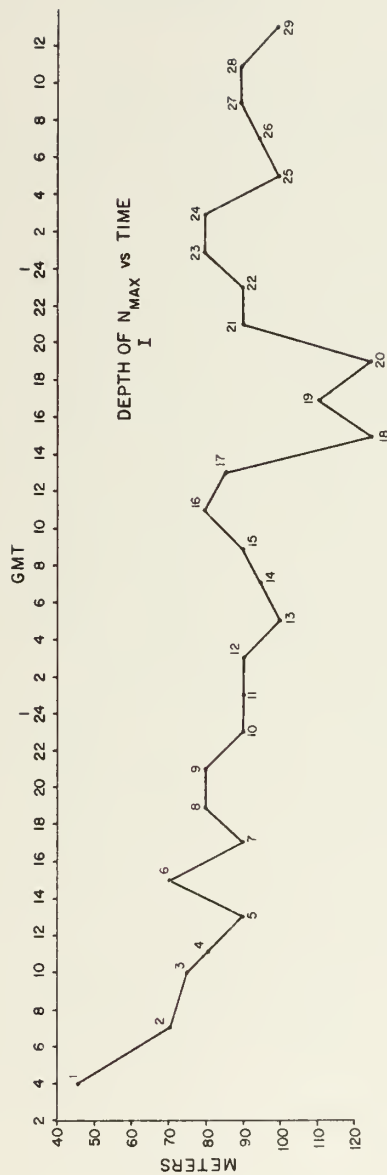
III.   RESULTS

A.   DISCUSSION

Results of the experiment discussed in this thesis lead to the
observation that the Vaisala-Brunt frequency in the Florida Current
near the axis of the current is a complex feature of stability with
no apparent periodicity with respect to depth or time. An examination
of each plot of frequency versus depth shows that the maximum fre-
quency occurs in the thermocline where the mean density gradient is
greatest and decreases above and below this level as the water becomes
more homogeneous. The smallest frequency values are observed in the
deeper layers and near the surface.

The weather during the data collection phase was particularly
stormy at the anchor station. This may account for the well mixed
upper layer and fairly abrupt thermoclines recorded. Thus, we ex-
perienced a three layer oceanic structure, with relatively homo-
geneous water above and below the thermocline layer. The plots of
frequency versus depth show a sharp maximum frequency at or just below
the thermocline boundary with smaller values of N elsewhere. The
thermocline thickness appears to be between 100-150 meters.

It is observed that the STD data provide continuous profiles of
the thermohaline, and therefore density, structure. The instrument
makes obsolete the older methods of interpolating between widely spaced
sample points. The STD reports discontinuities varying in magnitude,

time, and space as the rule rather than the exception. In the plots of frequency versus depth (Fig. 3), series of layers are observed clearly with sharp boundaries separating each from its neighboring layers, and these exist throughout the water column. The gradients or boundaries between the layers are often thin and sharp. The shallower layers show stronger lamination effects in the density structure than do the deeper depths. These are obvious observations of the data collected and are only the beginning of exploration into the recognition of this fine structure. Stommel and Fedorov (1967) present some interesting ideas on the dynamic processes involved in this fine structure and possible gradient effects on vertical mixing and mean turbulent transport. Data similar to that obtained for this thesis could be analyzed in the light of their studies and add much to the study of the Florida Current.

It is recognized that the Vaisala-Brunt frequency plays a major role in the study of internal waves, therefore its distribution with depth in the Florida Current is of interest. As noted, it is a very complicated function of depth. It appears that the complex structure occurs because of temperature and salinity perturbations.

The average plot of frequency versus depth presents a representative trace of the Vaisala-Brunt frequency for the Florida Current during the period of the cruise. The small deviations indicate that the average frequency figure might be useful in future calculations.

B.   RECOMMENDATIONS

In looking ahead to future work, it is recommended that this type cruise be conducted again in the summer months, preferably to obtain a seasonal variation of the density structure and see the effect on

the Vaisala-Brunt frequency. More emphasis should be placed into examining the fine structure, however, because this may lead to a better understanding of how the density structure varies, and why. It may also offer an understanding of the Vaisala frequency randomness of variation.

Project MIMI, a study of acoustic signal propagation across the Florida Straits, collected acoustic data simulaneously with our collection of density data. An examination of these data might provide an insight into the internal wave configurations of the Florida Current, or produce a means of measuring Vaisala-Brunt frequency directly, if a definite relationship exists between the variations of N and the acoustic variations received in Bimini. It offers an excellent opportunity to compare physical parameters with acoustic data taken simultaneously over an extended period, along the acoustic axis.

Any future investigations of the Vaisala-Brunt frequency in the Florida Current and its distribution will enhance the dynamical investigations of this region.

IV. SUMMARY

The Vaisala-Brunt frequency was calculated and investigated for a given location in the Florida Current near the axis of the current. The analysis of data from ninety-three separate STD stations gives a composite profile of the Vaisala-Brunt frequency versus depth for this region. The experiment was designed to obtain density information for an investigation of the fluctuations of the frequency as a function of time and depth. The results of the investigation are station profiles of the frequency versus depth, a mean profile for the aggregate of stations, a plot of maximum frequency versus time and the depth of N max versus time. The standard deviation and variance were computed at three selected depths for frequency, and for the depth of N max.

The investigation reveals that the frequency is a very complex function of depth and time which appears to vary in time, space, and magnitude in a random manner; typical of many oceanic processes. This randomness of the function is the most significant observation. A representative figure for N max is presented along with the depth at which it occurs for possible use in future work related to internal waves, acoustics, or density stability in the Florida Current.

CONCLUSIONS

The mean value of maximum Vaisala-Brunt frequency at this location in the Florida Current is $0.0169 \ sec^{-1}$ with a corresponding period of 6.19 minutes.

The average depth of occurrence of the maximum Vaisala-Brunt frequency is 100 meters with a standard deviation of $\pm$ 15.2 meters and a coefficient of variance of 16.6 per cent, or relative dispersion.

The standard deviation of the mean Vaisala-Brunt frequency at 100 meters is $\pm$ 0.0037 sec^{-1} with a coefficient of variance of 22.8 per cent. At 245 meters the standard deviation decreases to $\pm$ 0.0030 sec^{-1} and the coefficient of variance increases to 40.5 per cent. At 350 meters the standard deviation is $\pm$ 0.0031 sec^{-1} with a coefficient of variance of 60.9 per cent. All the coefficients of variance are increasing with depth, with the standard deviation fairly constant with depth. The deviation is slightly higher within the thermocline, near maximum frequency.

The average plot of frequency versus depth is shown in figure 4 and is representative of the Vaisala frequency in depth for this location.

The depth at which the maximum frequency occurs varies with time as does the magnitude of maximum frequency, however, no apparent periodicity is noticed. The randomness of this feature may be its most important characteristic.

LITERATURE CITED

BASCUAS, M.
      1968.   Cubic Spline interpolation (personal communication).

COX, C. S.
      1962.   The Seas, Vol. I, pp. 752-752. Wiley and Sons. New
                 York.

ECKART, C.
      1961.   Internal waves in the ocean. The Physics of Fluids,
                 Vol. 4, No. 7, pp. 791-798.

      1960.   Hydrodynamics of Oceans and Atmospheres. Permanon Press,
                 New York, pp. 55-71.

HAURWITZ, B., STOMMEL, H. and MUNK, W. H.
      1959.   On thermal unrest in the ocean. Rossby Memorial Volume,
                 pp. 74-94.

LaFOND, E. C., and LaFOND, K. G.
      1966.   Vertical and horizontal thermal structures in the sea.
                 NEL report 1395.

LEE, O. S.
      1959.   Observations on internal waves in shallow water. Journal
                 Acoustic Society of America, (8), pp. 312-321.

McLELLAN, H. J.
      1965.   Elements of Physical Oceanography. Pergamon Press,
                 New York.
PHILLIPS, O. M.
      1966.   Dynamics of the Upper Ocean. University Press, Cambridge.

POLLAK, M. J.
      1954.   Static stability parameters in oceanography. Journal
                 of Marine Research, (13), pp. 101-111.

RODEN, G. I.
      1968.   Spectral analysis and interpretation of salinity,
                 temperature, depth records. Journal of Geophysical
                 Research, Vol. 73, (2), pp. 535-539.

SPIEGEL, M. R.
      1961   Statistics. Schaum Publishing Co., New York.

STOMMEL, H., and FEDOROV, K. N.
      1967.   Small scale structure in temperature and salinity near
                 Timor and Mindanao. Tellus, Vol. 19, (2), pp. 306-325.

SVERDRUP, H. U., JOHNSON, M. W., and FLEMING, R. H.
      1942.   The Oceans. Prentice Hall, New York.

WENNEKENS, M. P.
1968.   Pressure measurements as related to the measurement of gradients in the ocean.   Journal of Ocean Technology, Vol. 2, No. 2, pp. 49-54.

APPENDIX A. VELOCITY OF SOUND TERM IN THE

VAISALA-BRUNT EQUATION

From Hook's Law: STRESS = K X STRAIN.

E = Bulk Modulus, in dimensions of pressure $\underline{FORCE}$ or $\underline{MASS \times ACCELERATION}$,
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ AREA $\qquad$ AREA

ρ = density, in dimensions of $\underline{MASS}$
$\qquad\qquad\qquad\qquad\qquad\qquad$ VOLUME

so that $\quad E = \dfrac{ML}{T^2L^2} = \dfrac{M}{LT^2}$ and $\rho = \dfrac{M}{L^3}$, $\hfill$ (1)

$\dfrac{E}{\rho} = \dfrac{M}{LT^2} \times \dfrac{L^3}{M} = \dfrac{L^2}{T^2} = \left(\dfrac{L}{T}\right)^2$ $\hfill$ (2)

which is in velocity units squared.

According to Newton, the speed of propagation of waves

$$c^2 = \frac{E}{\rho},$$ $\hfill$ (3)

substituting pressure P, for E, we obtain the same results, so that

$$\frac{P}{\rho} = c^2,$$ $\hfill$ (4)

and for an adiabatic change

$$\left(\frac{dP}{d\rho}\right)_A = c^2.$$ $\hfill$ (5)

APPENDIX B.   CUBIC SPLINE INTERPOLATION

A new approach based on an old idea is used to interpolate a set of data. It is based on the idea of using a different expression in each sub-interval $[X_i, X_{i+1}]$ and auxiliary conditions of smoothness at the joints instead of a simple closed form expression, valid over the entire interval. It was developed because the other methods (e.g., least squares) failed to give good results under certain conditions; at the two extremes of a set of data or could not provide a good evaluation of the first derivative along the computed polynomial fit.

The basis for all interpolation schemes is that given a discrete set of $n + 1$ points an nth degree polynomial can be passed. An interpolating polynomial of degree as high as the number of data points would allow can be obtained. But this can be impractical sometimes. Some of the problems encountered with interpolating schemes like the least square method are:

1. A set of data having points which are far apart from each other, will fail to give a good approximating polynomial, since the conditions for smoothness are very fragile, in most cases.

2. If the change in the slopes from point to point are very sharp, the resultant polynomial produces gross errors, since this rate of change of slopes has a direct effect, not only where the change occurs but also along the entire interval.

3. Theory states that the higher the degree of computed polynomial the better the approximation. Again, experience shows that one could go only so high on the degree of the interpolated polynomial before the errors

pile up making the whole thing frustrating, when a

considerable amount of data points are available.

Experience shows that polynomials higher than 13th

degree are impractical. But this depends on the

distribution of the data points.

All this led to the development of the Spline interpolation which

is based on the idea of passing a low degree polynomial through a few

points at a time making use of conditions of continuity and existence

of first and second derivative.

A cubic polynomial is interpolated in each sub interval $[X_i^-1, X_i]$,

i = 1,...n. It is accomplished with the following conditions met,

    1.   Continuity of the function

    2.   Continuity of first derivative

    3.   Continuity of second derivative.

Due to the nature of the computations, data which is equally spaced

provides short cuts to the procedure. With this in mind the data for

this experiment was chosen at equal intervals of depth.

For a mathematical treatment of the Cubic Spline Interpolation

scheme the reader is referred to "Approximation of Functions" by

Carl de Boor (1965). Elsevier Publishing Co., New York.

APPENDIX C.   STATISTICAL FORMULAS

MEAN FREQUENCY $\qquad \bar{X} = \dfrac{X}{n}$ (1)

$$n = \text{number of points}$$

STANDARD DEVIATION $\qquad S = \sqrt{\dfrac{X^2}{n} - (\bar{X})^2}$ (2)

VARIANCE $\qquad V = S^2$ (3)

COEFFICIENT OF VARIANCE (relative dispersion)

$$C = \frac{S}{\bar{X}} = \frac{\text{standard deviation}}{\text{mean}}$$ (4)

VITA

Lt. Joseph Paletta, Jr., USN was born in New York City, New York on November 15, 1937. His parents are Joseph Paletta and Lucrezia Mary Paletta. He received his elementary education in the Public School system of New York, and his secondary education at Stuyvesant High School, New York City, New York.

In September 1954 he enrolled at New York University's College of Engineering, Bronx, New York. In June 1956 he entered the U.S. Naval Academy, Annapolis, Maryland. Upon graduating in June 1960 with a B.S. degree, he was commissioned an Ensign in the U.S. Navy. He then represented the United States as a member of the 1960 Olympic fencing team in Rome. His subsequent naval service was in the Submarine service of the Atlantic Fleet on board the USS Sea Cat (SS399) and USS Tirante (SS420).

He was admitted to the Graduate School of the University of Miami in September, 1966. He was granted the degree of Master of Science in July 1968.

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